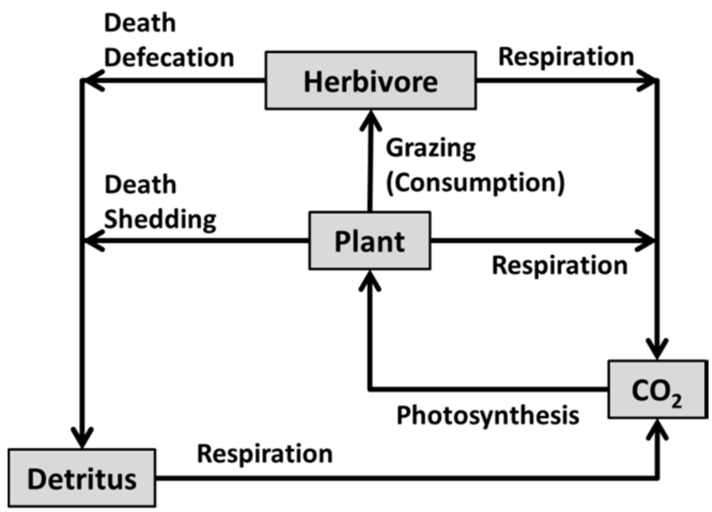
FW364
Midterm I - PRACTICE EXAM - KEY
February 2012

Mass Balance Practice Questions

1. You are a climate change modeler that is interested in how carbon flows through ecosystems. As a good ecosystem modeler, you start by developing a conceptual diagram of carbon flow. Draw a stock-and-flow model for carbon flow through an ecosystem with two trophic levels (plants and herbivores) and stocks for detritus and CO_2. Please label the model with words that describe each flow process.



2. Lake Michigan has a residence time of approximately 99 years and a volume of 4,900 km^3. Assuming the flow into Lake Michigan is the same as the flow out of Lake Michigan, what is the flow rate into Lake Michigan in units of km^3/yr? What is the flow rate in units of m^3/s? What is this assumption called that we are making in this problem?

T = S/F, so F = S/T
Given: **T = 99 yr**
S = 4,900 km^3
$F_i = F_o$

F_i = (4,900 km^3) / 99 yr = 49 km^3/yr

Converting from km^3/yr to m^3/yr:

$$49\ \frac{km^3}{yr} \times \frac{1000m}{1km} \times \frac{1000m}{1km} \times \frac{1000m}{1km} \times \frac{1yr}{365day} \times \frac{1day}{24hr} \times \frac{1hr}{60min} \times \frac{1min}{60s} = \mathbf{1554\ m^3/s}$$

If conversion was done using F_i with more decimals, the conversion yields 1569 m^3/s

3. Lake Linguini has a water residence time of 10 years, a surface area of 1 km^2, and an evaporation rate of 0.001 m/day. If precipitation rate is equal to evaporation rate and total inflow of water from the watershed (overland plus groundwater flow) to the lake is 100 m^3/day, what is the volume of the lake? You can assume there are other outflows from the lake for which we do not have data; you can also assume precipitation and overland plus groundwater flow are the only inputs.

Given: **Turnover time (T) = 10 years**
Lake surface area = 1 km^2
Evaporation rate (e) = 0.001 m/day = precipitation rate (p)
Watershed inflow (I) = 100 m^3/day

Assuming steady-state, we know that: T = S/F, so S = T x F, and $F_i = F_o$

Need to convert lake area from km^2 to m^2:

$$1km^2 \times \frac{1000m}{1km} \times \frac{1000m}{1km} = 1,000,000m^2$$

Need to find F_i, the total input of water to the lake:
Total precipitation input (P) = p x lake area = 0.001 m/day x 1,000,000 m^2 = 1000 m^3/day

F_i = P + I = 1000 m^3/day + 100 m^3/day = 1100 m^3/day = 401,500 m^3/yr

S = 10 yr x 401,500 m^3/yr = 4,015,000 m^3 = 0.00402 km^3

4. Continuing with Lake Linguini in Question 3, assume that the precipitation rate is twice the evaporation rate. What is the volume of the lake in this case?

Given: **Turnover time (T) = 10 years**
Lake surface area = 1 km^2
Precipitation rate (p) = Evaporation rate (e) x 2 = 0.002 m/day
Watershed inflow (I) = 100 m^3/day

Assuming steady-state, we know that: T = S/F, so S = T x F, and $F_i = F_o$

Need to convert lake area from km^2 to m^2:
1 km^2 = 1,000,000 m^2 (see above)

Need to find F_i, the total input of water to the lake:
Total precipitation input (P) = p x lake area = 0.002 m/day x 1,000,000 m^2 = 2000 m^3/day

F_i = P + I = 2000 m^3/day + 100 m^3/day = 2100 m^3/day = 766,500 m^3/yr

S = 10 yr x 766,500 m^3/yr = 7,665,000 m^3 = 0.0077 km^3

5. Fill in the blanks in the analogies below using the given word options (there are more word options available than you will use; some word options may be used more than once). Note: there will not be analogies on the test.

Word options:
Assimilation
Incorporation
Photosynthesis
Photosynthesis minus respiration
Respiration

Analogies:

Net primary production is to plants as ___**Incorporation**___ is to animals.

Assimilation is to animals as ___**Photosynthesis**__ is to plants.

Photosynthesis minus respiration is to plants as ___**Incorporation**___ is to animals.

6. A zebra population consisting of 10,000 individuals, each with a carbon content of 20 kg, consumes 10% of net primary production and has a carbon turnover time of 2 years. Zebra are herbivores and incorporate an average of 40% of what they consume. If the turnover time of plant biomass is 1 year, what is plant biomass in this ecosystem?

Given: Plant turnover time (T_p) = 1 year
Zebra turnover time (T_z) = 2 years
Zebra biomass (S_z) = 10,000 zebra x 20 kgC/zebra = 200,000 kgC

Assuming steady-state: T = S/F, so: S = T x F and F = S/T

Zebra input (F_z) = S_z/T_z = (200,000 kgC)/2 yr = 100,000 kgC/yr

Zebra consume 10% of net primary production (= plant input = F_p) and incorporate 40% of what they consume, so zebra incorporation (F_z) is 4% of F_p ($F_z = 0.04 \times F_p$)

Consequently, $F_p = F_z / 0.04$ = (100,000 kgC/yr) / 0.04 = 2,500,000 kgC/yr

We now know F_p and T_p, so can solve for S_p:

S_p = 1 yr x 2,500,000 kgC/yr = 2,500,000 kgC = 2.5×10^9 gC

Quantitative Tools Practice Questions

In the problems below, choose between each pair of model categories

7. The residence time equation: $T = S / F$, is an example of which type of model:

static or dynamic

deterministic or stochastic

8. The equation: $N_{t+1} = N_t \lambda_t$ is an example of which type of model:

static or **dynamic**

discrete or continuous

deterministic or **stochastic**

9. The equation: $N_{t+1} = N_t (1+ b' - d' - e') + I$ is an example of which type of model:

static or **dynamic**

discrete or continuous

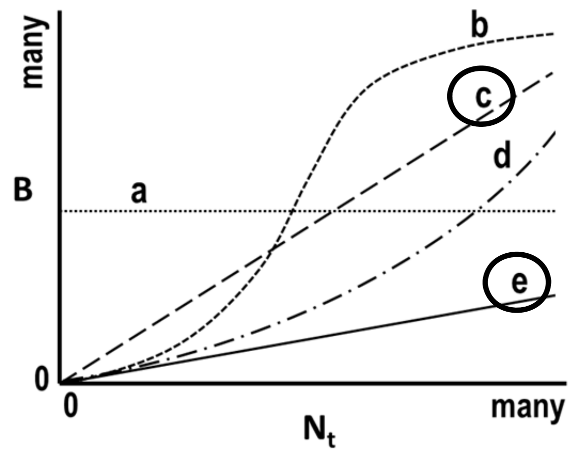
deterministic or stochastic

10. Which of the methods below could we use if we were interested in forecasting population size across multiple time periods for a population with immigration and emigration as described in the equation in Question 9?

obtain analytical solution or **use numerical simulation**

Population Growth Practice Questions

11. Below is a figure that shows five options for per capita birth rates (b'). If we define the number of births to be equal to the current population size multiplied by the per capita birth rate (i.e., B = b'N), which b' option(s) below could fit our equation for the number of births?



12. Starting from the simple observation that population growth between any two consecutive years for a population that reproduces annually is equal to the population size during the first year plus the number of individuals born less the number of individuals that die that year, derive the equation we use to forecast growth between consecutive time periods (express the growth rate in this equation using the finite population growth rate parameter, λ).

$\mathbf{N_{t+1} = N_t + B - D}$
$\mathbf{N_{t+1} = N_t + b'N_t - d'N_t}$
$\mathbf{N_{t+1} = N_t (1 + b' - d')}$

Let: $\mathbf{\lambda = 1 + b' - d'}$

Then: $\mathbf{N_{t+1} = N_t \lambda}$

13. From the equation you derived in Question 12, we can derive the equation $N_t = N_0 \lambda^t$ to forecast population growth at any time in the future from any time in the past. An important application of this equation is for determining the doubling time for a population, which can be expressed mathematically as the time when $N_t = 2N_0$. Beginning with the equation to forecast population growth ($N_t = N_0 \lambda^t$), derive the equation that we use to forecast the doubling time for any population exhibiting **discrete** growth.

Given: $N_t = N_0 \lambda^t$
$N_t = 2N_0$ **is expression for doubling time**

Rearranging expression for doubling time:

$$N_t = 2\,N_0 \rightarrow 2 = \frac{N_t}{N_0}$$

Rearranging forecasting equation:

$$N_t = N_0\lambda^t \rightarrow \frac{N_t}{N_0} = \lambda^t$$

Substituting expression for doubling time:

$$2 = \lambda^t$$

Take natural log (or $\log_{10}$) of both sides:

$$\ln(2) = \ln(\lambda^t) \rightarrow \ln(2) = t\ln(\lambda)$$

Rearrange to:

$$t_{doubling} = \frac{\ln(2)}{\ln(\lambda)} \quad \text{OR} \quad t_{doubling} = \frac{\log_{10}(2)}{\log_{10}(\lambda)}$$

14. How would the equation you derived in Question 13 change if we were interested in calculating quadrupling time?

Quadrupling time can be expressed as: $N_t = 4N_0$

Following the derivation above, $4 = \lambda^t$

Which continues on to yield:

$$t_{quadrupling} = \frac{\ln(4)}{\ln(\lambda)} \quad \text{OR} \quad t_{quadrupling} = \frac{\log_{10}(4)}{\log_{10}(\lambda)}$$

15. A bass population has a per capita birth rate of 5% per year and a per capita death rate from natural causes of 6% per year. Assuming that the population is closed, how long will it take for the population to double?

Given: **b' = 5% = 0.05**
d' = 6% = 0.06

Since rates given as percentages are finite rates, we must apply a discrete model of population growth to make this forecast:

$N_t = N_0 \lambda^t$, where λ is the finite population growth rate

In this problem, λ = 1 + b' – d' = 1 + 0.05 – 0.06 = 0.99

Since the net population growth rate, λ, is < 1, the population is declining, and therefore, the population will never double

16. For the bass population in Question 15, assume that population size is currently 1500 bass. If we harvest at a rate of 1.5% per year, how many bass will there be 10 years from now?

Given: **Harvest expressed as a finite rate: h' = 1.5% = 0.015**
b' and d' as above
N_0 = 1500 bass
Time (t) = 10 years

With harvest included, we need to redefine λ as:
λ = 1 + b' – d' – h' = 1 + 0.05 – 0.06 – 0.015 = 0.975

$N_t = N_0 \lambda^t$
N_t = 1500 bass * $(0.975)^{10}$
N_t = 1164 bass

17. We will now relax the assumption that the bass population in Questions 15-16 is a closed system. If bass leave the lake (emigrate) at a rate of 2% per year and a constant 200 bass are stocked each year, and the starting population size is 1500 bass (with the same birth, death, and harvest rates above), how many bass will be in the lake the following year?

Given: **Emigration as a finite rate: e' = 2% = 0.02**
Immigration as a constant addition: I = 200 bass per year
b', d', h', and N_0 as above
Time step (t) = 1 year

With a time step of only 1 year, need to use the equation for forecasting growth between consecutive years, $N_{t+1} = N_t \lambda$, as our starting point

As in Question 2 above, we need to redefine λ to include emigration:
λ = 1 + b' – d' – h' – e'

We can substitute the redefined λ into our growth equation, while also adding on a constant term for immigration:
$N_{t+1} = N_t$ (1 + b' – d' – h' – e') + I

Plugging in the givens:
N_{t+1} = 1500 bass (1 + 0.05 – 0.06 – 0.015 – 0.02) + 200 bass
N_{t+1} = 1633 bass

18. An elk population in Yellowstone National Park consists of 600 individuals and has an average population growth rate (λ) of 1.10 per year. Wolves have recently been re-introduced to the park as part of an overall effort to restore the Yellowstone ecosystem. The new wolf population is expected to kill about 5% of the elk population per year. Assuming that elk birth rate and rates of other types of mortality do not change in response to wolf predation, how many fewer elk will there be in 10 years as a result of wolf re-introduction? Is it reasonable to assume that wolves will not affect elk birth rate or mortality from other factors? Why or why not?

Given: **N_0 = 600 individuals**
λ = 1.10 in the absence of wolf predation
Elk death due to wolves as a finite rate: w' = 5% = 0.05

Since rates given as percentages are finite rates, we must apply a discrete model of population growth to make this forecast:
$N_t = N_0 \lambda^t$, where λ is the net population growth rate

Without wolves, $\lambda = 1 + b' - d' = 1.10$, where d' is natural death excluding wolves

Using the current net population growth rate of the elk, λ = 1.10, we can forecast the elk population size in 10 years in the absence of wolves as:
$N_t = N_0 \lambda^t$ = 600 elk x 1.10^{10} = 1556 elk

In the presence of wolves, we need to include in our model an additional mortality factor, wolf predation, which we can designate as w'. Since we are assuming that elk birth rate (b') and mortality from other factors (d') are not affected by wolves, and using our knowledge from above that: $1 + b' - d' = 1.10$, we can calculate the new population growth rate, λ, as:
$\lambda = (1 + b' - d') - w' = 1.10 - 0.05 = 1.05$

Using the latter value to project elk population size in 10 years:
$N_t = N_0 \lambda^t$ = 600 elk x 1.05^{10} = 977 elk

Number of elk without wolves – Number of elk with wolves = 1556 elk – 977 elk = 579 elk

<u>Answer: There will be 579 fewer elk in 10 years as a result of wolf predation</u>

However, it may not be reasonable to assume that wolves will not affect elk birth rate or mortality from other factors. We might expect elk birth rate (b') to increase after wolf re-introduction via density dependence, or wolves to kill some elk that were destined to die from other causes (i.e., d' may decrease). These two possibilities suggest that we may be overestimating the impact of wolves on elk numbers with our simple model.

19. Just last week I found a carton of eggnog in my refrigerator that I bought sometime in early December. Upon opening the container, there was undoubtedly a thriving bacteria population inside the carton. Given that bacteria grow **<u>continuously</u>** and have an instantaneous population growth rate of 0.60 per day, and assuming the initial population size was just a single bacterium, how many bacteria would be in my eggnog carton after sitting in the fridge for 60 days?

For this problem, we need to use the continuous growth rate equation: $N_t = N_0 e^{rt}$

Given: **Instantaneous population growth rate, r = 0.60 /day**
N_0 = 1 bacterium
Time = 60 days

<u>$N_t = N_0 e^{rt}$ = 1 bacterium * $e^{(0.60/\text{day} * 60 \text{ days})}$ = 4.3 x 10^{15} bacteria</u>

<u>Population Variation Practice Questions</u>

20. Bluegill can be found in both ponds and lakes. In which of these locations would **<u>demographic</u>** stochasticity have a greater effect on bluegill population growth? Assume there are many more bluegill that can live in a lake than in a pond.

    a. <u>Pond population</u>
    b. Lake population
    c. Demographic stochasticity would have a similar effect on both populations

21. In which population would **<u>environmental</u>** stochasticity have a greater effect?

    a. Pond population
    b. Lake population
    c. <u>Environmental stochasticity would have a similar effect on both populations</u>

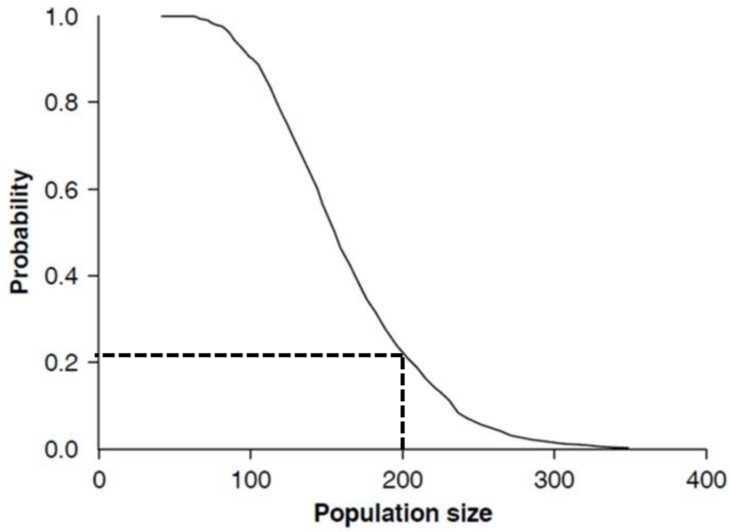
22. The table below shows the results from 500 replications of a stochastic model that forecasted population size of bottlenose dolphins from this year to next year. Across all 500 replications, 71 individuals was the smallest number forecasted. Using the data below, fill out the remaining two columns of the table (*cumulative number of trials* and *probability of decline*).

Population size	Number of trials	**Cumulative number of trials**	**Probability of decline to N_c**
71	2	**2**	**0.004**
72	3	**5**	**0.010**
73	7	**12**	**0.024**
74	16	**28**	**0.056**
75	36	**64**	**0.128**
76	74	**138**	**0.276**
77	165	**303**	**0.606**

23. Assume that a management goal for the dolphin population in Question 22 is to keep the population size greater than 75 individuals because smaller populations are too likely to go extinct. What is the probability that this management goal will **<u>not</u>** be met? (i.e., what is the probability that the dolphin population size will be **75 or fewer** dolphins next year?)

    <u>From the table above, probability of 75 or fewer dolphins next year = 0.128 = 12.8%</u>

24. The figure below is an explosion risk curve for muskox on Nunivak Island (Figure 2.7 in the textbook). What is the approximate probability that the population will explode to 200 or more muskox?



<u>The probability is just above 0.2 ≈ 20%</u>